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Introduction

In this book, I offer a refresher on some basic maths operations, such as addition, subtraction, multiplication and division, before moving on to the more advanced topic of algebra. So let me introduce you to algebra. This introduction is somewhat like what would happen if I were to introduce you to my friend Donna. I'd say, 'This is Donna. Let me tell you something about her.' After giving a few well-chosen tidbits of information about Donna, I'd let you ask more questions or fill in more details. In this book, you find some well-chosen topics and information, and I try to fill in details as I go along.

As you read this introduction, you're probably in one of two situations:

- You've taken the plunge and bought the book.
- You're checking things out before committing to the purchase.

In either case, you'd probably like to have some good, concrete reasons why you should go to the trouble of reading and finding out about algebra.

One of the most commonly asked questions in a mathematics classroom is, 'What will I ever use this for?' Some teachers can give a good, convincing answer. Others hem and haw and stare at the floor. My favourite answer is, 'Algebra gives you *power*.' Algebra gives you the power to move on to bigger and better things in mathematics. Algebra gives you the power of knowing that you know something that your neighbour doesn't know. Algebra gives you the power to be able to help someone else with an algebra task or to explain to others these logical mathematical processes.

Algebra is a system of symbols and rules that is universally understood, no matter what the spoken language. Algebra provides a clear, methodical process that can be followed from beginning to end. It's an organisational tool that is most useful when followed with the appropriate rules. What power! Some people like algebra because it can be a form of puzzle-solving. You solve a puzzle by finding the value of a variable. You may prefer Sudoku or crosswords, but it wouldn't hurt to give algebra a chance, too.

About This Book

This book isn't like a mystery novel; you don't have to read it from beginning to end. In fact, you can peek at how it ends and not spoil the rest of the story.

I divide the book into some general topics — from the beginning nuts and bolts to the important tool of factoring to equations, applications and geometry. So you can dip into the book wherever you want, to find the information you need.

Throughout the book, I use many examples, each a bit different from the others, and each showing a different twist to the topic. The examples have explanations to aid your understanding. (What good is knowing the answer if you don't know how to get the right answer yourself?)

The vocabulary I use is mathematically correct *and* understandable. So whether you're listening to your teacher or talking to someone else about algebra, you'll be speaking the same language.

Along with the *how*, I show you the *why*. Sometimes remembering a process is easier if you understand why it works and don't just try to memorise a meaningless list of steps.

I don't use many conventions in this book, but you should be aware of the following:

- When I introduce a new term, I put that term in *italics* and define it nearby (often in parentheses).
- I express numbers or numerals either with the actual symbol, such as 8, or the written-out word: *eight*. Operations, such as +, are either shown as this symbol or written as *plus*. The choice of expression all depends on the situation — and on making it perfectly clear for you.

The *sidebars* (those little grey boxes) are interesting but not essential to your understanding of the text. If you're short on time, you can skip the sidebars. Of course, if you read them, I think you'll be entertained.

Foolish Assumptions

I don't assume that you're as crazy about maths as I am — and you may be even *more* excited about it than I am! I do assume, though, that you have a mission here — to brush up on your basic skills, improve your maths grade, or just have some fun. I also assume that you have some experience with algebra — for example, full exposure for a year or so.

You may remember the first time algebra came up in your maths class. I can distinctly remember my first algebra teacher, Miss McDonald, saying, ‘This is an *n*.’ My whole secure world of numbers was suddenly turned upside down. I hope your first reaction was better than mine.

Wherever you are in your maths journey, or what aspect you need to improve on, never fear. Help is here!

Icons Used in This Book

The little drawings in the margin of the book are there to draw your attention to specific text. Here are the icons I use in this book:

ALGEBRA RULES

To make everything work out right, you have to follow the basic rules of algebra (or mathematics in general). You can’t change or ignore them and arrive at the right answer. Whenever I give you an algebra rule, I mark it with this icon.

REMEMBER

Paragraphs marked with the Remember icon help clarify a symbol or process. I may discuss the topic in another section of the book, or I may just remind you of a basic algebra rule that I discuss earlier.

TIP

The Tip icon isn’t life-or-death important, but it generally can help make your life easier — at least your life in maths and algebra.

WARNING!

The Warning icon alerts you to something that can be particularly tricky. Errors crop up frequently when working with the processes or topics next to this icon, so I call special attention to the situation so you won’t fall into the trap.

Where to Go From Here

If you want to refresh your basic skills or boost your confidence, start with Part I. If you’re ready to jump into the guts of algebra, or looking for some factoring practice and need to pinpoint which method to use with what, go to Part II. Part III is for you if you're ready to solve equations; you can find just about any type you're ready to attack. Part IV is where the good stuff is — applications and geometry — things to do with all those good solutions. The list in Part V is usually what you'd look at after visiting one of the other parts, but why not start there? It's a fun place!

Studying more advanced maths and algebra can give you some logical exercises, and thinking logically can help you with all aspects of life — at school and afterwards.

The best *why* for studying algebra is just that it's beautiful. Yes, you read that right. Algebra is poetry, deep meaning and artistic expression. Just look and you'll find it. Also, don't forget that it gives you *power*.

Enjoy the adventure!

Chapter 2

Working with Whole Numbers

In This Chapter

- Reviewing addition
- Understanding subtraction
- Viewing multiplication as a fast way to do repeated addition
- Getting clear on division

When most people think of maths, the first thing that comes to mind is four little (or not-so-little) words: Addition, subtraction, multiplication and division. I call these operations the main four basic operations.

In this chapter, I introduce you (or reintroduce you) to these little gems. Although I assume you're already familiar with these four operations, this chapter reviews them, taking you from what you may have missed to what you need to succeed as you move onward and upward in maths.

Adding Things Up

Addition is the first operation you find out about, and it's almost everybody's favourite. It's simple, friendly and straightforward. No matter how much you worry about maths, you've probably never lost a minute of sleep over addition. Addition is all about bringing things together, which is a positive goal. For example, suppose you and I are standing in line to buy tickets for a movie. I have $30 and you have only $10. I could lord it over you and make you feel crummy that I can go to the movies and you can't. Or instead, you and I can join forces, adding together my $30 and your $10 to make $40. Now, not only can we both see the movie, but we may even be able to buy some popcorn, too.

Addition uses only one sign — the plus sign (+): Your equation may read 2 + 3 = 5, 12 + 2 = 14 or 27 + 44 = 71, but the plus sign always means the same thing.

REMEMBER

When you add two numbers together, those two numbers are called *addends*, and the result is called the *sum*. So in the first example, the addends are 2 and 3, and the sum is 5.

In line: Adding larger numbers in columns

When you want to add larger numbers, stack them on top of each other so that the ones digits line up in a column, the tens digits line up in another column, and so on. Then add column by column, starting from the ones column on the right. Not surprisingly, this method is called *column addition*. Here's how you add 55 + 31 + 12. First add the ones column:

$$\begin{array}{r} 5\mathbf{5} \\ 3\mathbf{1} \\ +1\mathbf{2} \\ \hline \mathbf{8} \end{array}$$

Next, move to the tens column:

$$\begin{array}{r} \mathbf{5}5 \\ \mathbf{3}1 \\ +\mathbf{1}2 \\ \hline \mathbf{9}8 \end{array}$$

This problem shows you that 55 + 31 + 12 = 98.

Carry on: Dealing with two-digit answers

Sometimes when you're adding a column, the sum is a two-digit number. In that case, you need to write down the ones digit of that number and carry the tens digit over to the next column to the left — that is, write this digit above the column so you can add it with the rest of the numbers in that column. For example, suppose you want to add 376 + 49 + 18. In the ones column, 6 + 9 + 8 = 23, so write down the 3 and carry the 2 over to the top of the tens column:

$$\begin{array}{r} 2 \\ 37\mathbf{6} \\ 4\mathbf{9} \\ +\ 1\mathbf{8} \\ \hline \mathbf{3} \end{array}$$

Now continue by adding the tens column. In this column, 2 + 7 + 4 + 1 = 14, so write down the 4 and carry the 1 over to the top of the hundreds column:

$$\begin{array}{r} 1\mathbf{2} \\ 3\mathbf{7}6 \\ \mathbf{4}9 \\ +\ \mathbf{1}8 \\ \hline \mathbf{4}3 \end{array}$$

Continue adding in the hundreds column:

$$\begin{array}{r} \mathbf{1}2 \\ \mathbf{3}76 \\ 49 \\ +\ 18 \\ \hline \mathbf{4}43 \end{array}$$

This problem shows you that 376 + 49 + 18 = 443.

This process applies no matter how large the numbers themselves become. The following example shows the steps for adding two five-digit numbers — 12,495 + 14,821.

$$\begin{array}{r} {\scriptstyle 1\ \ 1}\quad\ \ \\ 12{,}495 \\ +14{,}821 \\ \hline 27{,}316 \\ \hline \end{array}$$

Here's the problem broken down:

1. **Add the first column.**

   In this example, 5 + 1 = 6, so you write **6** in the answer space.

2. **Add the second column.**

   Here, 9 + 2 = 11, so you write the **1** in the answer space and carry the 1 above the 4.

3. **Continue adding the columns, from right to left.**

   So, in the next column, 1 + 4 + 8 = 13, so you write the **3** in the answer space and carry the 1 above the 2.

   Moving to the next column, 1 + 2 + 4 = 7, so you write **7** in the answer space. Finally, 1 + 1 = 2, so you write **2** in the answer space.

So 12,495 + 14,821 = 27,316.

This process also applies if you have more than two or three numbers to add, as the following example shows. Remember you can add zeros to fill the empty spaces if that helps keep the digits aligned.

$$
\begin{array}{r}
^{1}^{1}{}^{1} \\
23{,}562 \\
65{,}321 \\
00{,}567 \\
+\ 00{,}015 \\
\hline
89{,}465 \\
\hline
\end{array}
$$

Here's how you work through the problem:

1. **Add the first column.**

   In this example, 2 + 1 + 7 + 5 = 15, so you write the **5** in the answer space and carry the 1 above the 6.

2. **Add the second column.**

   Here, 1 + 6 + 2 + 6 + 1 = 16, so you write the **6** in the answer space and carry the 1 above the 5.

3. **Continue adding the columns, from right to left.**

   So, in the next column, 1 + 5 + 3 + 5 + 0 = 14, so you write the **4** in the the answer space and carry the 1 above the 3.

   Next, 1 + 3 + 5 + 0 + 0 = 9, so you write **9** in the answer space.

   Finally, 2 + 6 + 0 + 0 = 8, so you write **8** in the answer space.

TIP

Using grid paper can help you line up the digits correctly and carry them into the correct position.

Take It Away: Subtracting

Subtraction is usually the second operation you discover, and it's not much harder than addition. Still, there's something negative about subtraction — it's all about who has more and who has less. Suppose you and I have been running on treadmills at the gym. I'm happy because I ran 3 kilometres, but then you start bragging that you ran 10 kilometres. You subtract and tell me that I should be very impressed that you ran 7 kilometres farther than I did. (But with an attitude like that, don't be surprised if you come back from the showers to find your running shoes filled with liquid soap!)

As with addition, subtraction has only one sign: the minus sign (–). You end up with equations such as 4 – 1 = 3, and 14 – 13 = 1, and 93 – 74 = 19.

When you subtract one number from another, the result is called the *difference*. This term makes sense when you think about it: When you subtract, you find the difference between a higher number and a lower one.

In subtraction, the first number is called the *minuend* and the second number is called the *subtrahend*. But almost nobody ever remembers which is which, so when I talk about subtraction, I prefer to say *the first number* and *the second number*.

One of the first facts you probably heard about subtraction is that you can't take away more than you start with. In that case, the second number can't be larger than the first. And if the two numbers are the same, the result is always 0. For example, 3 – 3 = 0; 11 – 11 = 0; and 1,776 – 1,776 = 0. Later someone breaks the news that you *can* take away more than you have. When you do, though, you need to place a minus sign in front of the difference to show that you have a negative number, a number below 0:

4 – 5 = –1

10 – 13 = –3

88 – 99 = –11

When subtracting a larger number from a smaller number, remember the words *switch* and *negate*: You *switch* the order of the two numbers and do the subtraction as you normally would, but at the end, you *negate* the result by attaching a minus sign. For example, to find 10 – 13, you switch the order of these two numbers, giving you 13 – 10, which equals 3; then you negate this result to get –3. That's why 10 – 13 = –3.

The minus sign does double duty, so don't get confused. When you stick a minus sign between two numbers, it means the first number minus the second number. But when you attach it to the front of a number, it means that this number is a negative number.

I also go into more detail on negative numbers and the main four operations in Chapter 3.

Columns and stacks: Subtracting larger numbers

To subtract larger numbers, stack one on top of the other as you do with addition. (For subtraction, however, don't stack more than two numbers — put the larger number on top and the smaller one underneath it.) For example, suppose you want to subtract 386 – 54. To start, stack the two numbers and begin subtracting in the ones column: 6 – 4 = 2:

$$\begin{array}{r} 38\mathbf{6} \\ -5\mathbf{4} \\ \hline \mathbf{2} \end{array}$$

Next, move to the tens column and subtract 8 – 5 to get 3:

$$\begin{array}{r} 3\mathbf{8}6 \\ -\mathbf{5}4 \\ \hline \mathbf{3}2 \end{array}$$

Finally, move to the hundreds column. This time, 3 – 0 = 3:

$$\begin{array}{r} \mathbf{3}86 \\ -54 \\ \hline \mathbf{3}32 \end{array}$$

This problem shows you that 386 – 54 = 332.

Can you spare a ten? Borrowing to subtract

Sometimes the top digit in a column is smaller than the bottom digit in that column. In that case, you need to borrow from the next column to the left. Borrowing is a two-step process:

1. **Subtract 1 from the top number in the column directly to the left.**

   Cross out the number you're borrowing from, subtract 1, and write the answer above the number you crossed out.

2. **Add 10 to the top number in the column you were working in.**

For example, suppose you want to subtract 386 – 94. The first step is to subtract 4 from 6 in the ones column, which gives you 2:

$$\begin{array}{r} 38\mathbf{6} \\ -9\mathbf{4} \\ \hline \mathbf{2} \end{array}$$

When you move to the tens column, however, you find that you need to subtract 8 – 9. Because 8 is smaller than 9, you need to borrow from the hundreds column. First, cross out the 3 and replace it with a 2, because 3 – 1 = 2:

$$\begin{array}{r} \mathbf{2} \\ \cancel{\mathbf{3}}86 \\ -94 \\ \hline 2 \end{array}$$

Next, place a 1 in front of the 8, changing it to an 18, because 8 + 10 = 18:

$$\begin{array}{r} 2 \\ \cancel{3}\mathbf{18}6 \\ -94 \\ \hline 2 \end{array}$$

Now you can subtract in the tens column: 18 – 9 = 9:

$$\begin{array}{r} 2\mathbf{18}6 \\ -\mathbf{9}4 \\ \hline \mathbf{9}2 \end{array}$$

The final step is simple: 2 – 0 = 2:

$$\begin{array}{r} \mathbf{2}186 \\ -94 \\ \hline \mathbf{2}92 \end{array}$$

Therefore, 386 – 94 = 292.

In some cases, the column directly to the left may not have anything to lend. Suppose, for instance, that you want to subtract 1,002 – 398. Beginning in the ones column, you find that you need to subtract 2 – 8. Because 2 is smaller than 8, you need to borrow from the next column to the left. But the digit in the tens column is a 0, so you can't borrow from there because the cupboard is bare, so to speak:

$$\begin{array}{r} 10\mathbf{0}2 \\ -39\mathbf{8} \\ \hline \end{array}$$

REMEMBER

When borrowing from the next column isn't an option, you need to borrow from the nearest non-zero column to the left.

In this example, the column you need to borrow from is the thousands column. First, cross out the 1 and replace it with a 0. Then place a 1 in front of the 0 in the hundreds column:

$$\begin{array}{cccc} \mathbf{0} & & & \\ \cancel{1} & \mathbf{1}0 & 0 & 2 \\ & -3 & 9 & 8 \end{array}$$

Now cross out the 10 and replace it with a 9. Place a 1 in front of the 0 in the tens column:

$$\begin{array}{cccc} 0 & \mathbf{9} & & \\ \cancel{1} & \cancel{\mathbf{10}} & \mathbf{1}0 & 2 \\ & -3 & 9 & 8 \end{array}$$

Finally, cross out the 10 in the tens column and replace it with a 9. Then place a 1 in front of the 2:

$$\begin{array}{cccc} 0 & 9 & \mathbf{9} & \\ \cancel{1} & \cancel{10} & \cancel{\mathbf{10}} & \mathbf{12} \\ & -3 & 9 & \mathbf{8} \end{array}$$

At last, you can begin subtracting in the ones column: 12 – 8 = 4:

$$\begin{array}{rrrr} 0 & 9 & 9 & \\ \cancel{1} & \cancel{10} & \cancel{10} & \mathbf{12} \\ & -3 & 9 & \mathbf{8} \\ \hline & & & \mathbf{4} \end{array}$$

Then subtract in the tens column: 9 – 9 = 0:

$$\begin{array}{rrrr} 0 & 9 & \mathbf{9} & \\ \cancel{1} & \cancel{10} & \cancel{10} & 12 \\ & -3 & \mathbf{9} & 8 \\ \hline & & \mathbf{0} & 4 \end{array}$$

Then subtract in the hundreds column: 9 – 3 = 9:

$$\begin{array}{rrrr} 0 & 9 & \mathbf{9} & \\ \cancel{1} & \cancel{10} & 10 & 12 \\ & -3 & \mathbf{9} & 8 \\ \hline & \mathbf{6} & \mathbf{0} & 4 \end{array}$$

Because nothing is left in the thousands column, you don't need to subtract anything else. Therefore, 1,002 – 398 = 604.

This process applies no matter how large the numbers themselves become, as the following example shows. (Remember that you can add zeros to fill in spaces at the front of the number to help keep everything aligned.)

In this example, you need to work out 1,609,452 – 413,651. Note that a zero has been added to the number being subtracted, so that there are the same number of digits in each number.

$$\begin{array}{r} {}^{5\,10\;8}\quad{}^{14}\;\; \\ 1,\cancel{609},\cancel{4}52 \\ -0,413,651 \\ \hline 1,195,801 \end{array}$$

Here's how to break down the problem:

1. **Subtract the numbers in the first column.**

   In this example, 2 – 1 = 1, so you write **1** in the answer space.

2. **Subtract the numbers in the second column.**

   Here, 5 – 5 = 0, so you write **0** in the answer space.

3. **Continue subtracting the columns, moving from right to left.**

   In the next column, you can't take 6 from 4 so you need to borrow from the 9. The 4 becomes 14, and the subtraction can then be completed — $14 - 6 = 8$, so you can write **8** in the answer space.

   Next, $8 - 3 = 5$, so write **5** in the answer space.

   You can't take 1 from 0 so you need to borrow from the 6. The 0 becomes 10, the subtraction can then be completed — $10 - 1 = 9$, so write **9** in the answer space.

   In the next column, $5 - 4 = 1$, so write **1** in the answer space.

   Finally, $1 - 0 = 1$, so write **1** in the answer space.

So $1{,}609{,}452 - 413{,}651 = 1{,}195{,}801$.

Multiplying

Multiplication is often described as a sort of shorthand for repeated addition. For example:

4×3 means add 4 to itself 3 times: $4 + 4 + 4 = 12$

9×6 means add 9 to itself 6 times: $9 + 9 + 9 + 9 + 9 + 9 = 54$

100×2 means add 100 to itself 2 times: $100 + 100 = 200$

Although multiplication isn't as warm and fuzzy as addition, it's a great timesaver. For example, suppose you play in a junior cricket team, and you've just won a game against the toughest team in the league. As a reward, your coach promised to buy three pies for each of the nine players on the team. To find out how many pies your coach needs, you can add 3 together 9 times. Or you can save time by multiplying 3 times 9, which gives you 27. Therefore, you need 27 pies (plus a whole lot of tomato sauce).

REMEMBER

When you multiply two numbers, the two numbers that you're multiplying are called *factors*, and the result is the product.

In multiplication, the first number is also called the *multiplicand* and the second number is the *multiplier.* But almost nobody ever remembers — or uses — these words.

Signs of the times

When you're first introduced to multiplication, you use the times sign (×). As you move onward and upward on your math journey, you need to be aware of the conventions I discuss in the following sections.

The symbol · is sometimes used to replace the symbol ×. For example,

$$\begin{array}{ccc} 4 \cdot 2 = 8 & \text{means} & 4 \times 2 = 8 \\ 6 \cdot 7 = 42 & \text{means} & 6 \times 7 = 42 \\ 53 \cdot 11 = 583 & \text{means} & 53 \times 11 = 583 \end{array}$$

In Part I of this book, I stick to the tried-and-true symbol × for multiplication. Just be aware that the symbol · exists so that you won't be stumped if your teacher or textbook uses it.

In maths beyond arithmetic, using parentheses without another operator stands for multiplication. The parentheses can enclose the first number, the second number, or both numbers. For example,

$$\begin{array}{ccc} 3(5) = 15 & \text{means} & 3 \times 5 = 15 \\ (8)7 = 56 & \text{means} & 8 \times 7 = 56 \\ (9)(10) = 90 & \text{means} & 9 \times 10 = 90 \end{array}$$

This switch makes sense when you stop to consider that the letter *x*, which is often used in algebra, looks a lot like the multiplication sign ×. So in this book, when I start using *x* in Part II, I also stop using × and begin using parentheses without another sign to indicate multiplication.

Memorising the multiplication table

You may consider yourself among the multiplicationally challenged. That is, you consider being called upon to remember 9 × 7 a tad less appealing than being dropped from an airplane while clutching a parachute purchased from the boot of some guy's car. If so, this section is for you.

Looking at the old multiplication table

One glance at the old multiplication table, Table 2-1, reveals the problem. If you saw the movie *Amadeus*, you may recall that Mozart was criticised for writing music that had 'too many notes'. Well, in my humble opinion, the multiplication table has too many numbers.

Table 2-1 The Monstrous Standard Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

I don't like the multiplication table any more than you do. Just looking at it makes my eyes glaze over. With 100 numbers to memorise, no wonder so many people just give up and carry a calculator.

Introducing the short multiplication table

If the multiplication table from Table 2-1 were smaller and a little more manageable, I'd like it a lot more. So here's my short multiplication table, in Table 2-2.

Table 2-2 The Short Multiplication Table

	3	*4*	*5*	*6*	*7*	*8*	*9*
3	9	12	15	18	21	24	27
4		16	20	24	28	32	36
5			25	30	35	40	45
6				36	42	48	54
7					49	56	63
8						64	72
9							81

As you can see, I've gotten rid of a bunch of numbers. In fact, I've reduced the table from 100 numbers to 28. I've also shaded 11 of the numbers I've kept.

Is just slashing and burning the sacred multiplication table wise? Is it even legal? Well, of course it is! After all, the table is just a tool, like a hammer. If a hammer's too heavy to pick up, you need to buy a lighter one. Similarly, if the multiplication table is too big to work with, you need a smaller one. Besides, I've removed only the numbers you don't need. For example, the condensed table doesn't include rows or columns for 0, 1, or 2. Here's why:

- Any number multiplied by 0 is 0 (people call this trait the *zero property of multiplication*).
- Any number multiplied by 1 is that number itself (which is why mathematicians call 1 the *multiplicative identity* — because when you multiply any number by 1, the answer is identical to the number you started with).
- Multiplying by 2 is fairly easy; if you can count by 2s — 2, 4, 6, 8, 10 and so forth — you can multiply by 2.

The rest of the numbers I've gotten rid of are redundant. (And not just redundant, but also repeated, extraneous and unnecessary!) For example, any way you slice it, 3×5 and 5×3 are both 15 (you can switch the order of the factors because multiplication is *commutative*). In my condensed table, I've simply removed the clutter.

So what's left? Just the numbers you need. These numbers include a grey row and a grey diagonal. The grey row is the 5 times table, which you probably know pretty well. (In fact, the 5s may evoke an early-childhood memory of running to find a hiding place on a warm spring day while one of your friends counted in a loud voice: 5, 10, 15, 20 . . .)

The numbers on the grey diagonal are the square numbers — when you multiply any number by itself, the result is a square number. You probably know these numbers better than you think.

Getting to know the short multiplication table

In about an hour, you can make huge strides in memorising the multiplication table. To start, make a set of flash cards that give a multiplication problem on the front and the answer on the back. They may look like Figure 2-1.

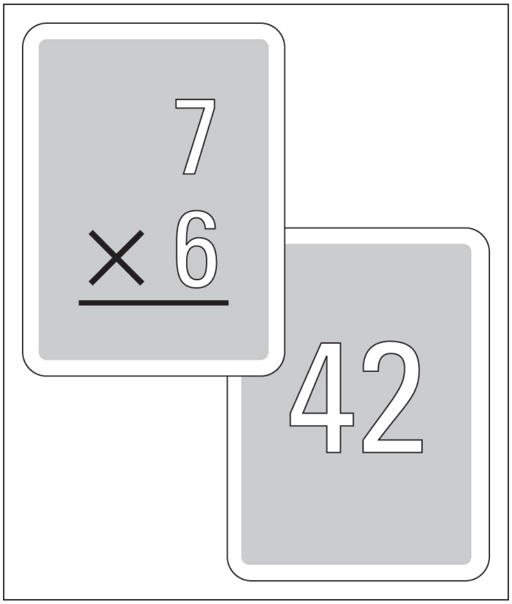

Figure 2-1: Both sides of a flash card, with 7×6 on the front and 42 on the back.

Remember, you need to make only 28 flash cards — one for every example in Table 2-2. Split these 28 into two piles — a 'grey' pile with 11 cards and a 'white' pile with 17. (You don't have to colour the cards grey and white; just keep track of which pile is which, according to the shading in Table 2-2.) Then begin:

1. **5 minutes:** Work with the grey pile, going through it one card at a time. If you get the answer right, put that card on the bottom of the pile. If you get it wrong, put it in the middle so you get another chance at it more quickly.
2. **10 minutes:** Switch to the white pile and work with it in the same way.
3. **15 minutes:** Repeat Steps 1 and 2.

Now take a break. Really — the break is important to rest your brain. Come back later in the day and do the same thing.

When you're done with this exercise, you should find going through all 28 cards with almost no mistakes to be fairly easy. At this point, feel free to make cards for the rest of the standard times table — you know, the cards with all the 0, 1 and 2 times tables on them and the redundant problems — mix all 100 cards together, and amaze your family and friends.

TIP

To the nines: A slick trick

Here's a trick to help you remember the 9 times table. To multiply any one-digit number by 9:

1. **Subtract 1 from the number being multiplied by 9 and jot down the answer.**

   For example, suppose you want to multiply 7×9. Here, $7 - 1 = \mathbf{6}$.

2. **Jot down a second number so that, together, the two numbers you wrote add up to 9. You've just written the answer you were looking for.**

3. **Adding, you get $\mathbf{6} + \mathbf{3} = \mathbf{9}$.** So $7 \times 9 = 63$.

As another example, suppose you want to multiply 8×9:

$8 - 1 = \mathbf{7}$

$7 + \mathbf{2} = 9$

So $8 \times 9 = 72$.

This trick works for every one-digit number except 0 (but you already know that $0 \times 9 = 0$).

Double digits: Multiplying larger numbers

The main reason to know the multiplication table is so you can more easily multiply larger numbers. For example, suppose you want to multiply 53×7. Start by stacking these numbers on top of one another with a line underneath, and then multiply 3 by 7. Because $3 \times 7 = 21$, write down the 1 and carry the 2:

$$\begin{array}{r} \mathbf{2} \\ 5\mathbf{3} \\ \times\mathbf{7} \\ \hline \mathbf{1} \end{array}$$

Next, multiply 7 by 5. This time, $5 \times 7 = 35$. But you also need to add the 2 that you carried over, which makes the result 37. Because 5 and 7 are the last numbers to multiply, you don't have to carry, so write down the 37 — you find that $53 \times 7 = 371$:

$$\begin{array}{r} 2 \\ \mathbf{5}3 \\ \times\,\mathbf{7} \\ \hline \mathbf{37}1 \end{array}$$

When multiplying larger numbers, the idea is similar. For example, suppose you want to multiply 53 by 47. (The first few steps — multiplying by the 7 in 47 — are the same, so I pick up with the next step.) Now you're ready to multiply by the 4 in 47. But remember that this 4 is in the tens column, so it really means 40. So to begin, put a 0 directly under the 1 in 371:

$$\begin{array}{r} 53 \\ \times\mathbf{47} \\ \hline 371 \\ \mathbf{2}0 \end{array}$$

This 0 acts as a placeholder so that this row is arranged properly.

REMEMBER

When multiplying by larger numbers with two digits or more, use one placeholding zero when multiplying by the tens digit, two placeholding zeros when multiplying the hundreds digit, three zeros when multiplying by the thousands digit, and so forth.

Now you multiply 3×4 to get 12, so write down the 2 and carry the 1:

$$\begin{array}{r} \mathbf{1} \\ 5\mathbf{3} \\ \times\mathbf{47} \\ \hline 371 \\ \mathbf{2}0 \end{array}$$

Continuing, multiply 5×4 to get 20, and then add the 1 that you carried over, giving a result of 21:

$$\begin{array}{r} \mathbf{1} \\ \mathbf{5}3 \\ \times\mathbf{47} \\ \hline 371 \\ \mathbf{21}20 \end{array}$$

To finish, add the two products (the multiplication results):

$$\begin{array}{r} 53 \\ \times 47 \\ \hline 371 \\ +\ 2120 \\ \hline \mathbf{2{,}491} \end{array}$$

So $53 \times 47 = 2{,}491$.

Multiplication of larger numbers is performed in the same way as the multiplication of smaller numbers, with grid paper being even more useful.

REMEMBER

You may find you move to working out larger multiplication problems on a calculator. As you do so, the communicative nature of multiplication is the most important thing to keep in mind — that is, that $4 \times 6 = 6 \times 4$. This rule is very useful when you reach the more complicated algebraic multiplication in later high school.

Doing Division Lickety-Split

The last of the Big Four operations is division. Division literally means splitting things up. For example, suppose you're on a picnic with two friends. You've brought along 12 cheese sticks as snacks, and want to split them fairly so that you and your two friends each get the same number (don't want to cause a fight, right?).

You each get four cheese sticks. This problem tells you that

$$12 \div 3 = 4$$

As with multiplication, division also has more than one sign: The division sign (÷) and the fraction slash (/) or fraction bar (—). So some other ways to write the same information are

$$12/3 = 4 \text{ and } \frac{12}{3} = 4$$

Whichever way you write it, the idea is the same: When you divide 12 cheese sticks equally among three people, each person gets 4 of them.

REMEMBER

When you divide one number by another, the first number is called the *dividend*, the second is called the *divisor* and the result is the *quotient*. For example, in the division from the earlier example, the dividend is 12, the divisor is 3 and the quotient is 4.

Whatever happened to the division table?

Considering how much time teachers spend on the multiplication table, you may wonder why you've never seen a division table. For one thing, the multiplication table focuses on multiplying all the one-digit numbers by each other. This focus doesn't work too well for division because division usually involves at least one number that has more than one digit.

Besides, you can use the multiplication table for division, too, by reversing the way you normally use the table. For example, the multiplication table tells you that $6 \times 7 = 42$. You can reverse this equation to give you these two division problems:

$$42 \div 6 = 7$$

$$42 \div 7 = 6$$

Using the multiplication table in this way takes advantage of the fact that multiplication and division are *inverse operations.*

Making short work of long division

In the olden days, knowing how to divide large numbers — for example, 62,997 ÷ 843 — was important. People used *long division*, an organised method for dividing a large number by another number. The process involved dividing, multiplying, subtracting and dropping numbers down.

But face it — one of the main reasons the pocket calculator was invented was to save 21st-century humans from ever having to do long division again.

Having said that, I need to add that your teacher and maths-crazy friends may not agree. Perhaps they just want to make sure you're not completely helpless if your calculator disappears somewhere into your backpack or the Bermuda Triangle. But if do you get stuck doing page after page of long division against your will, you have my deepest sympathy.

I will go this far, however: Understanding how to do long division with some not-too-horrible numbers is a good idea. In this section, I give you a good start with long division, telling you how to do a division problem that has a one-digit divisor.

Recall that the *divisor* in a division problem is the number that you're dividing by. When you're doing long division, the size of the divisor is your main concern: Small divisors are easy to work with, and large ones are a royal pain.

Working through an example

Suppose you want to find 860 ÷ 5. Start off by writing the problem like this:

$$5\overline{)860}$$

Unlike the other main four operations, long division moves from left to right. In this case, you start with the number in the hundreds column (8). To begin, ask how many times 5 goes into 8 — that is, what's 8 ÷ 5? The answer is 1 (with a little bit left over), so write 1 directly above the 8. Now multiply 1 × 5 to get 5, place the answer directly below the 8, and draw a line beneath it:

$$\begin{array}{r@{}l} & \;1 \\ 5 & \overline{)860} \\ & \;\underline{5} \end{array}$$

Subtract 8 – 5 to get 3. (***Note:*** After you subtract, the result should always be smaller than the divisor. If not, you need to write a higher number above the division symbol.) Then bring down the 6 to make the new number 36:

$$\begin{array}{r@{}l} & \;1 \\ 5 & \overline{)860} \\ & \underline{-5} \\ & \;\;36 \end{array}$$

These steps are one complete cycle — to complete the problem, you just need to repeat them. Now ask how many times 5 goes into 36 — that is, what's 36 ÷ 5? The answer is 7 (with a little left over). Write 7 just above the 6, and then multiply 7 × 5 to get 35; write the answer under 36:

$$\begin{array}{r@{}l} & \;17 \\ 5 & \overline{)860} \\ & \underline{-5} \\ & \;\;36 \\ & \underline{-35} \end{array}$$

Now subtract to get 36 – 35 = 1; bring down the 0 next to the 1 to make the new number 10:

$$\begin{array}{r}
172 \\
5\overline{)860} \\
\underline{-5} \\
36 \\
\underline{-35} \\
10
\end{array}$$

Another cycle is complete, so begin the next cycle by asking how many times 5 goes into 10 — that is, 10 ÷ 5. The answer this time is 2. Write down the 2 in the answer above the 0. Multiply to get 2 × 5 = 10, and write this answer below the 10:

$$\begin{array}{r}
172 \\
5\overline{)860} \\
\underline{-5} \\
36 \\
\underline{-35} \\
10 \\
\underline{-10}
\end{array}$$

Now subtract 10 – 10 = 0. Because you have no more numbers to bring down, you're finished, and here's the answer (that is, the quotient):

$$\begin{array}{r}
172 \\
5\overline{)860} \\
\underline{-5} \\
36 \\
\underline{-35} \\
10 \\
\underline{-10} \\
0
\end{array}$$

So 860 ÷ 5 = 172.

This problem divides evenly, but many don't. If the number does not divide equally it can be written as a remainder or as a fraction or decimal.

Division of larger numbers is performed in the same way as the division of smaller numbers and, again, using grid paper can help you keep track of columns.

As you move to completing larger division problems on a calculator, still keep the long division process in mind. It comes in handy when you reach some of the more complicated algebraic division problems in later high school.

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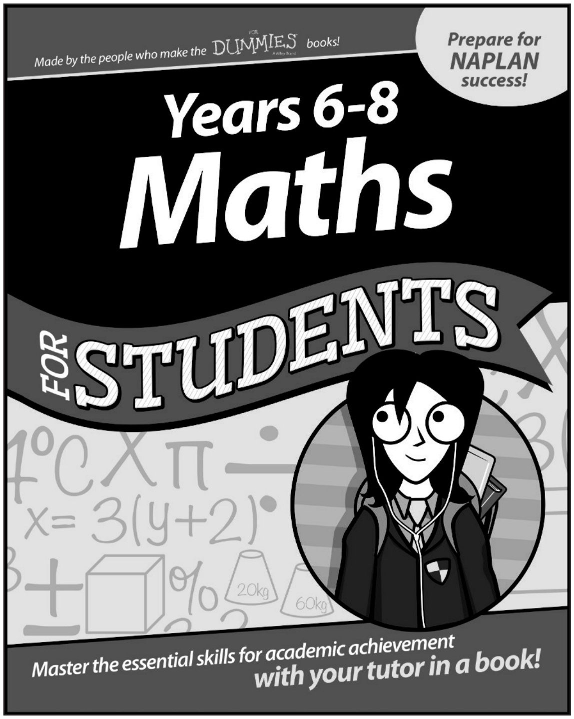

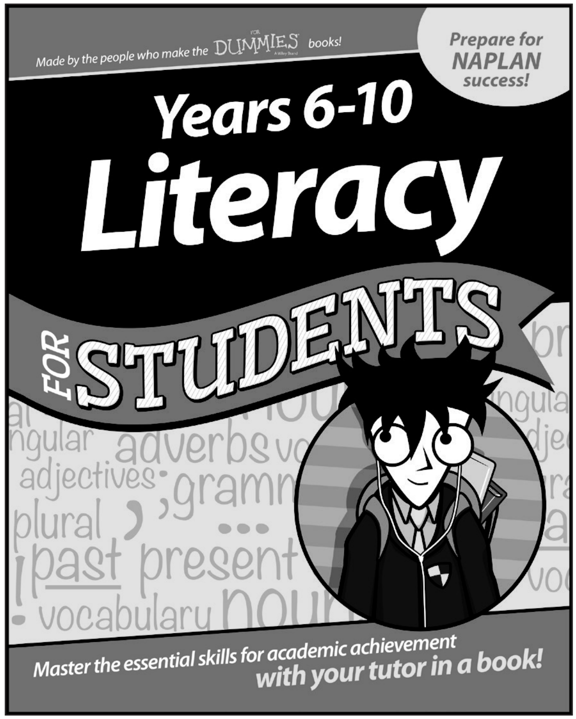

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